JEE Advanced 2026

Sample Paper - 3 (Paper-1)

Time Allowed: 3 hours **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and four sub-sections as below.

MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

NUM

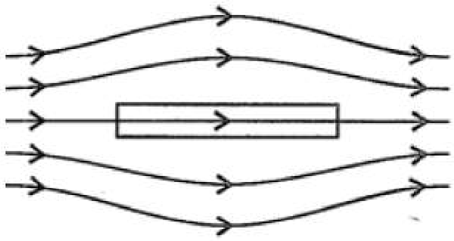
- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

MATCH

- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- You will get +4 marks for the correct response and -1 for the incorrect response.

Physics

1. The given figure represents a material which is: **[3]**



a) Trimagnetic b) ferromagnetic

c) diamagnetic

d) paramagnetic

2. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be Δp and $1.5\ \Delta p$ respectively. Then [3]

a) $3\ m_A = 2\ m_B$

b) $2\ m_A = 3\ m_B$

c) $9\ m_A = 4\ m_B$

d) $4\ m_A = 9\ m_B$

3. The acceleration experienced by a moving boat after its engine is cut-off, is given by: $a = -kv^3$, where k is a constant. If v_0 is the magnitude of velocity at cut-off, then the magnitude of the velocity at time t after the cut-off is: [3]

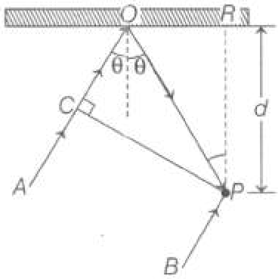
a) $\frac{v_0}{1+2ktv_0^2}$

b) $\frac{v_0}{2ktv_0^2}$

c) $\frac{v_0}{\sqrt{1-2ktv_0^2}}$

d) $\frac{v_0}{\sqrt{1+2ktv_0^2}}$

4. In the adjacent diagram, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition of θ for constructive interference at P between the ray BP and reflected ray OP [3]



a) $\cos\theta = \frac{\lambda}{4d}$

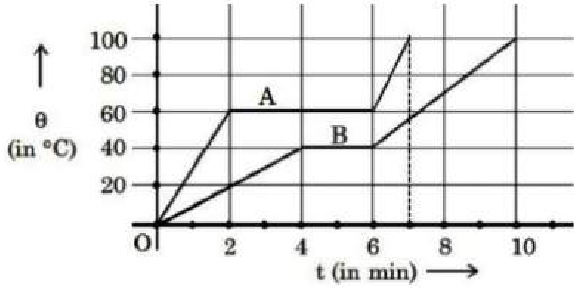
b) $\sec\theta - \cos\theta = \frac{\lambda}{d}$

c) $\cos\theta = \frac{3\lambda}{2d}$

d) $\sec\theta - \cos\theta = \frac{4\lambda}{d}$

5. Two solids A and B of equal mass are heated at a constant rate under identical conditions. Their temperature θ as a function of time t is given in figure. Then [4]

choose the correct statement(s):



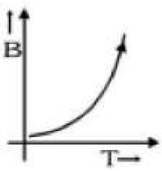
a) The latent heats of the fusion of the solids A and B are in the ratio 2 : 1.

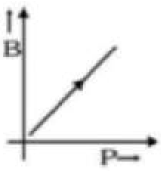
b) The specific heat of each solid at the melting point is finite.

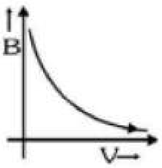
c) The ratio of melting points of the solids A and B is 3 : 2.

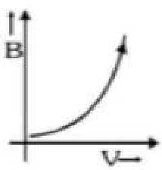
d) The ratio of the specific heats of the solids A and B is 1 : 3.

6. A sample of gas follows process represented by PV^2 = constant. Bulk modulus for this process is B , then which of the following graph is correct? **[4]**

a) 

b) 

c) 

d) 

7. A transistor is used in the common emitter mode as an amplifier. Then **[4]**

a) the input signal is connected in series with the voltage applied to bias the base-collector junction

b) the input signal is connected in series with the voltage applied to bias the base-emitter junction

c) the base-emitter junction is forward-biased

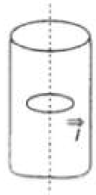
d) the base-emitter junction is reverse-biased

8. A stationary source emits sound of frequency f_0 = 492 Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat **[4]**

frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received)

9. A screw gauge having 100 equal divisions and a pitch of length 1 mm is used to measure the diameter of a wire of length 5.6 cm. The main scale reading is 1 mm and the 47^{th} circular division coincides with the main scale. Find the curved surface area of the wire in cm^2 to an appropriate significant figure. (use $\pi = \frac{22}{7}$) **[4]**

10. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire loop of resistance 0.005 Ω and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. **[4]**



The current varies as $I = I_0 \cos 300t$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300t)$, then N is

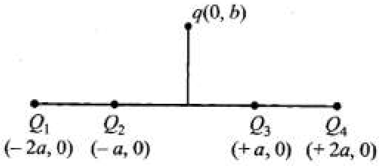
11. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0\ J\ mol^{-1}\ K^{-1}$, the decrease in its internal energy, in joule, is ________. **[4]**

12. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6. If the acceleration of the truck is 5 m/s^2, the frictional force acting on the block is ________ newtons. **[4]**

13. In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance C μF across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take rms values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is ϕ. **[4]**
Assume, $\pi\sqrt{3} \approx 5$.
The value of C is ________.

14. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at x = - 2a, - a, + a and + 2a, respectively. A positive charge q is placed on the positive y axis at a distance b > 0. Four options of the signs of these charges are given in List-I. The direction of the forces on the charge q is given in List-II. Match List-I with **[4]**

List-II and select the correct answer using the code given below the lists



List I	List II
(P) Q_1, Q_2, Q_3, Q_4 all positive	**(1)** +x
(Q) Q_1, Q_2 positive; Q_3, Q_4 negative	**(2)** - x
(R) Q_1, Q_4 positive; Q_2, Q_3 negative	**(3)** + y
(S) Q_1, Q_3 positive Q_2, Q_4 negative	**(4)** - y

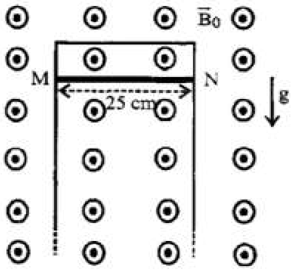
a) P - 4, Q - 2, R - 3, S - 1

b) P - 3, Q - 1, R - 4, S - 2

c) P - 4, Q - 2, R - 1, S - 3

d) P - 3, Q - 1, R - 2, S - 4

15. A thin conducting rod MN of mass 20 gm, length 25 cm and resistance 10 Ω is held on frictionless, long, perfectly conducting vertical rails as shown in the figure. There is a uniform magnetic field $B_0 = 4$ T directed perpendicular to the plane of the rod-rail arrangement. The rod is released from rest at time t = 0 and it moves down along the rails. Assume air drag is negligible. Match each quantity in List - I with an appropriate value from List -II, and choose the correct option. [Given: The acceleration due to gravity $g = 10\ ms^{-2}$ and $e^{-1} = 0.4$] **[4]**



List - I	List - II
(P) At t = 0.2 s, the magnitude of the induced emf in Volt	(1) 0.07
(Q) At t = 0.2 s, the magnitude of the magnetic force in Newton	(2) 0.14
(R) At t = 0.2 s, the power dissipated as heat in Watt	(3) 1.20
(S) The magnitude of terminal velocity of the rod in ms^{-1}	(4) 0.12
	(5) 2.00

a) (P) → (4), (Q) → (3), (R) → (1), (S) → (2)

b) (P) → (3), (Q) → (4), (R) → (2), (S) → (5)

c) (P) → (3), (Q) → (1), (R) → (4), (S) → (5)

d) (P) → (5), (Q) → (2), (R) → (3), (S) → (1)

16. Match **List I** with **List II**: **[4]**

List I	**List II**
(A) Isothermal Process	(I) Work done by the gas decreases internal energy
(B) Adiabatic Process	(II) No change in internal energy
(C) Isochoric Process	(III) The heat absorbed goes partly to increase internal energy and partly to do work
(D) Isobaric Process	(IV) No work is done on or by the gas

a) (A) - I, (B) - II, (C) - IV, (D) - III

b) (A) - II, (B) - I, (C) - III, (D) - IV

c) (A) - II, (B) - I, (C) - IV, (D) - III

d) (A) - I, (B) - II, (C) - III, (D) - IV

Chemistry

17. What is DDT among the following? **[3]**

a) Non-biodegradable pollutant

b) A fertilizer

c) Greenhouse gas

d) Biodegradable pollutant

18. The value of $\log_{10} K$ for a reaction $A \rightleftharpoons B$ is **[3]**
(Given : $\Delta_r H^o_{298K}$ = -54.07 kJ mol^{-1}, $\Delta_r S^\circ_{298K}$ = 10 JK^{-1} mol^{-1} and R = 8.314 JK^{-1} mol^{-1}; 2.303 × 8.314 × 298 = 5705)

a) 95

b) 100

c) 5

d) 10

19. The energy of an electron in the first Bohr orbit of H-atom is -13.6 eV. The possible energy value(s) of the excited state(s) for electrons in Bohr orbits of hydrogen is (are) **[3]**

a) +6.8 eV

b) -6.8 eV

c) -4.2 eV

d) -3.4 eV

20. In the electrolytic cell, flow of electrons is from **[3]**

a) Anode to cathode through internal supply

b) Cathode to anode in solution

c) Cathode to anode through external supply

d) Cathode to anode through internal supply

21. The correct option(s) regarding the com plex $[Co(en)(NH_3)_3(H_2O)]^{3+}$ (en^- = $H_2NCH_2CH_2NH_2$) is (are) **[4]**

a) It will have three geometrical isomers if bidentate 'en' is

b) It is paramagnetic

replaced by two cyanide ligands

c) It has two geometrical isomers

d) It absorbs light at longer wavelength as compared to $[Co(en)(NH_3)_4]^{3+}$

22. Resonance structures of a molecule should have: **[4]**

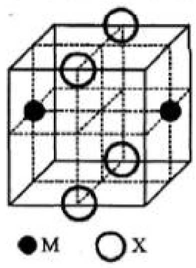
a) the same number of paired electrons

b) identical arrangement of atoms

c) identical bonding

d) nearly the same energy content

23. The cubic unit cell structure of a compound containing cation M and anion X is shown below. When compared to the anion, the cation has smaller ionic radius. Choose the correct statement(s). **[4]**



a) The cation M and anion X have different coordination geometries.

b) The ratio of M-X bond length to the cubic unit cell edge length is 0.866.

c) The empirical formula of the compound is MX.

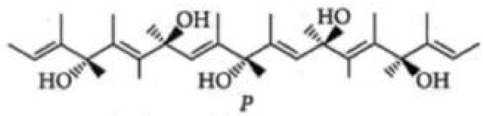
d) The ratio of the ionic radii of cation M to anion X is 0.414.

24. Consider the sulphides HgS, PbS, CuS, Sb_2S_3, As_2S_3 and CdS. Number of these sulphides soluble in 50% HNO_3 is ________. **[4]**

25. The stoichiometric reaction of 516 g of dimethyldichlorosilane with water results in a tetrameric cyclic product X in 75% yield. The weight (in g) of X obtained is ________. **[4]**

[Use, molar mass ($g\ mol^{-1}$): H = 1, C = 12,0 = 16, Si = 28, Cl = 35.5]

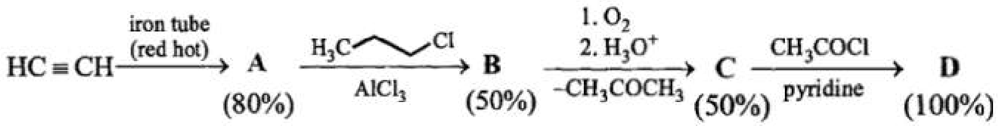
26. The total number of chiral molecules formed from one molecule of P on complete ozonolysis (O_3, Zn/H_2O) is ________. **[4]**



27. The mole fraction of a solute in a solution is 0.1. At 298 K, molarity of this solution is the same as its molality. Density of this solution at 298 K is 2.0 g cm^{-3}. The ratio of the molecular weights of the solute and solvent, $\left(\frac{MW_{solute}}{MW_{solvent}}\right)$ is **[4]**

28. The difference in the oxidation numbers of the two types of sulphur atoms in $Na_2S_4O_6$ is: **[4]**

29. If the reaction sequence given below is carried out with 15 moles of acetylene, the amount of the product D formed (in g) is ________. **[4]**



The yields of A, B, C and D are given in parentheses.
[Given: Atomic mass of H = 1, C = 12, O = 16, Cl = 35]

30. Match List I with List II. **[4]**

List I Type of Hydride	List II Example
(A) Electron deficient hydride	(I) MgH_2
(B) Electron rich hydride	(II) HF
(C) Electron precise hydride	(III) B_2H_6
(D) Saline hydride	(IV) CH_4

a) A - III, B - II, C - IV, D - I
b) A - II, B - III, C - I, D - IV
c) A - II, B - III, C - IV , D - I
d) A - III, B - II, C - I, D - IV

31. Match List-I with List-II : **[4]**

List-I Name of oxo acid	List-II Oxidation state of P
(A) Hypophosphorous acid	(I) +5
(B) Orthophosphoric acid	(II) +4
(C) Hypophosphoric acid	(II) +3

(D) Orthophosphorous acid	(Iv) +2
	(V) +1

a) (A) - (IV), (B) - (V), (C) - (II), (D) - (III)

b) (A) - (IV), (B) - (I), (C) - (II), (D) - (III)

c) (A) - V), (B) - (IV), (C) - (II), (D) - (III)

d) (A) - (V), (B) - (I), (C) - (II), (D) - (III)

32. Match the following drugs with their therapeutic actions: **[4]**

(i) Ranitidine	(A) Antidepressant
(ii) Nardil (Phenelzine)	(B) Antibiotic
(iii) Chloramphenicol	(C) Antihistamine
(iv) Dimetane (Brompheniramine)	(D) Antacid
	(E) Analgesic

a) (i) - (E); (ii) - (A); (iii) - (C); (iv) - (D)

b) (i) - (D); (ii) - (C); (iii) - (A); (iv) - (E)

c) (i) - (D); (ii) - (A); (iii) - (B); (iv) - (C)

d) (i) - (A); (ii) - (C); (iii) - (B); (iv) - (E)

Maths

33. Considering only the principal values of the inverse trigonometric functions, the value of $\tan\left(\sin^{-1}\left(\frac{3}{5}\right) - 2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$ is **[3]**

a) $\frac{7}{24}$

b) $\frac{-5}{24}$

c) $\frac{-7}{24}$

d) $\frac{5}{24}$

34. If P (1, 2), Q (4, 6), R (5, 7) and S(a, b) are the vertices of a parallelogram PQRS, then **[3]**

a) a = 3, b = 4

b) a = 2, b = 4

c) a = 2, b = 3

d) a = 3, b = 5

35. Three randomly chosen non-negative integers x, y and z are found to satisfy the equation x + y + z = 10. Then the probability that z is even, is **[3]**

a) $\frac{1}{2}$

b) $\frac{36}{55}$

c) $\frac{5}{11}$

d) $\frac{6}{11}$

36. A line with positive direction cosines passes through the point P (2, -1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals [3]

a) $\sqrt{3}$

b) 2

c) 1

d) $\sqrt{2}$

37. Let a ∈ R and let f : R → R be given by $f(x) = x^5 - 5x + a$. Then [4]

a) f(x) has three real roots if a > 4

b) f(x) has three real roots if a < -4

c) f(x) has three real roots if -4 < a < 4

d) f(x) has only real root if a > 4

38. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x. Then [4]

a) h is increasing whenever f is decreasing

b) h is increasing whenever f s increasing

c) h is decreasing whenever f is decreasing

d) nothing can be said in general.

39. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then [4]

a) $b_0 = 0, b_1 = n$

b) $b_0 = 0, b_1 = n^2 - 3n + 3$

c) $b_0 = -1, b_1 = n$

d) $b_0 = 1, b_1 = 3$

40. Let z be a complex number with non-zero imaginary part. If $\frac{2+3z+4z^2}{2-3z+4z^2}$ is a real number, then the value of $|z|^2$ is ________. [4]

41. In a triangle ABC, let A B = $\sqrt{23}$, BC = 3 and CA = 4. Then the value of $\frac{\cot A+\cot C}{\cot B}$ is [4]

42. The number of 4-digit integers in the closed interval [2022, 4482] formed by using the digits 0, 2, 3, 4, 6, 7 is ________. [4]

43. The positive integer value of n > 3 satisfying the equation **[4]**

$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

44. A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0, -\alpha)$ to the parabola $x^2 = -4ay$ where $a > 0$. Let L be the line passing through $(0, -\alpha)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB. If $r : s = 1 : 16$, then the value of $24a$ is ________. **[4]**

45. Let k be a positive real number and let **[4]**

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If det (adj A) + det (adjB) = 10^6, then [k] is equal to

46. Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3×3 matrix $M = (a_{ij})_{3\times3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for $i = 1, 2, 3$ and and $j = 1, 2, 3$. **[4]**

Match each entry in List-I to the correct entry in List-II.

	List-I		List-II
(P)	The number of matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j, is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $C_j = 0$ for all j, is	(2)	12
(R)	Let $M = (a_{ij})_{3\times3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then the number of elements in the set $\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R}, M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix} \right\}$ is	(3)	infinite
(S)	Let $M = (a_{ij})_{3\times3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then the absolute value of the determinant of M is	(4)	6
		(5)	0

a) $(P) \rightarrow (2), (Q) \rightarrow (4), (R) \rightarrow (1), (S) \rightarrow (5)$ b) $(P) \rightarrow (2), (Q) \rightarrow (4), (R) \rightarrow (3), (S) \rightarrow (5)$

c) $(P) \rightarrow (1), (Q) \rightarrow (5), (R) \rightarrow (3), (S) \rightarrow (4)$ d) $(P) \rightarrow (4), (Q) \rightarrow (2), (R) \rightarrow (5), (S) \rightarrow (1)$

47. Match List I with List II and select the correct answer using the code given below the lists: **[4]**

List-I	**List-II**
(P) Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c})$ and $2(\vec{c} \times \vec{a})$ is	(1) 100
(Q) Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a} + \vec{b}), 3(\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is	(2) 30
(R) Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$	(3) 24
(S) Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is	(4) 60

a) P → 1, Q → 4, R → 3, S → 2 b) P → 4, Q → 2, R → 3, S → 1

c) P → 3, Q → 4, R → 1, S → 2 d) P → 2, Q → 3, R → 1, S → 4

48. Match the Following: **[4]**

List-I	**List-II**
(P) The number of polynomials f(x) with non-negative integer coefficients of degree ≤ 2, satisfying f(0) = 0 and $\int_0^1 f(x)dx = 1$, is	(1) 8
(Q) The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	(2) 2
(R) $\int_{-2}^{2} \frac{3x^2}{(1+e^x)} dx$ equals	(3) 4

(S) $\dfrac{\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}{\left(\int_{0}^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}$	(4) 0

a) (P) - (2), (Q) - (3), (R) - (4), (S) - (1)

b) (P) - (3), (Q) - (2), (R) - (4), (S) - (1)

c) (P) - (3), (Q) - (2), (R) - (1), (S) - (4)

d) (P) - (2), (Q) - (3), (R) - (1), (S) - (4)

Solution

Physics

1.

(c) diamagnetic

Explanation:

diamagnetic

2. **(a)** 3 m_A = 2 m_B

Explanation:

Process is isothermal. Therefore, T = constatn. $\left(p \propto \frac{1}{V}\right)$ volume is increasing, therefore, pressure will decrease.

In chamber A $\rightarrow$

$-\Delta p=\left(p_A\right)_i-\left(p_A\right)_f=\frac{n_A R T}{V}-\frac{n_A R T}{2 V}$

$=\frac{n_A R T}{2 V}$ ...(i)

In chamber B $\rightarrow$

$-1.5 \Delta p=\left(p_B\right)_i-\left(p_B\right)_f=\frac{n_B R T}{V}-\frac{n_B R T}{2 V}$

$=\frac{n_B R T}{2 V}$ ...(ii)

From Eqs. (i) and (ii)

$\frac{n_A}{n_B}=\frac{1}{1.5}=\frac{2}{3}$ or $\frac{m_A / M}{m_B / M}=\frac{2}{3}$

or $\frac{m_A}{m_B}=\frac{2}{3}$ or 3 m_A = 2 m_B

3.

(d) $\frac{v_0}{\sqrt{1+2 k t v_0^2}}$

Explanation:

$\frac{v_0}{\sqrt{1+2 k t v_0^2}}$

4. **(a)** $\cos \theta=\frac{\lambda}{4 d}$

Explanation:

PR = d

$\therefore$ PO = d secθ

and CO = PO cos 2θ = d sec θ cos 2θ

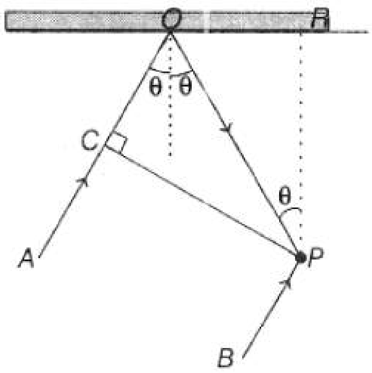
path difference between the two rays is,

Δx = PO + OC = (d sec θ + d sec θ cos 2 θ)

phase difference between the two rays is

$\Delta \phi=\pi$ (one is reflected, while another is direct)

Therefore, the condition for the constructive interface should be



$\Delta x=\frac{\lambda}{2},\frac{3\lambda}{2}....$

or $d\sec\theta(1+\cos 2\theta)=\frac{\lambda}{2}$

or $\left(\frac{d}{\cos\theta}\right)\left(2\cos^2\theta\right)=\frac{\lambda}{2}$ or $\cos\theta=\frac{\lambda}{4d}$

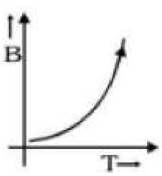
5. **(a)** The latent heats of the fusion of the solids A and B are in the ratio 2 : 1.

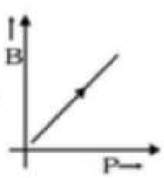
(c) The ratio of melting points of the solids A and B is 3 : 2.

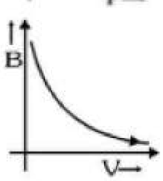
(d) The ratio of the specific heats of the solids A and B is 1 : 3.

Explanation: Correct Statement is-

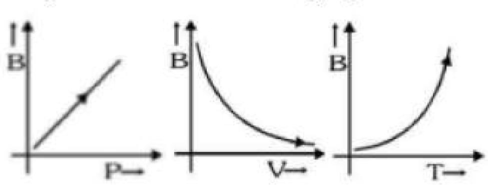
- The ratio of the specific heats of the solids A and B is 1 : 3
- The ratio of melting points of the solids A and B is 3 : 2.
- The latent heats of the fusion of the solids A and B are in the ratio 2 : 1.

6. **(a)** 

(b) 

(c) 

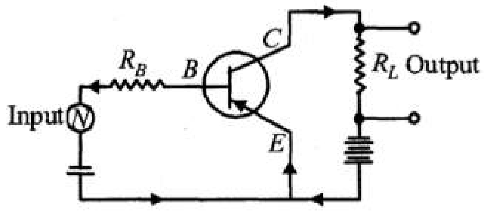
Explanation: These graph are correct-



7. **(b)** the input signal is connected in series with the voltage applied to bias the base-emitter junction

(c) the base-emitter junction is forward-biased

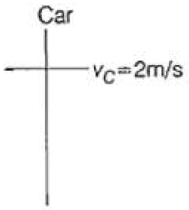
Explanation:



The circuit for a p-n-p transistor used in the common emitter mode as an amplifier is shown in figure. The base (B) emitter (E) junction is forward-biased and the input signal is connected in series with the voltage applied to bias the base emitter junction.

8. 6

Explanation:



Frequency observed at car

$f_1 = f_0\left(\frac{v+v_C}{v}\right)$ (v = speed of sound)

Frequency of reflected sound as observed at the source

$f_2 = f_1\left(\frac{v}{v-v_C}\right) = f_0\left(\frac{v+v_C}{v-v_C}\right)$

Beat frequency = f_2 - f_0

$= f_0\left[\frac{v+v_C}{v-v_C} - 1\right] = f_0\left[\frac{2v_C}{v-v_C}\right]$

$= 492 \times \frac{2\times2}{328} = 6$ Hz

9. 2.6

Explanation:

Least count, L.C. = $\frac{1mm}{100}$ = 0.01 mm

Diameter = MSR + CSR × (L.C.) = 1 mm + 47 × (0.01) mm = 1.47 mm

Curved surface area = $2\pi rl = 2\pi\frac{D}{2}l = \pi Dl$

$= \frac{22}{7} \times 1.47 \times 56\ mm^2 = 2.58724\ cm^2$

$= 2.6\ cm^2$ (Rounding off to two significant figures)

10. 6

Explanation:

Take the circular tube as a long solenoid. The wires are closely wound. Magnetic field inside the solenoid is

$B = \mu_0 ni$

Here, n = number of turns per unit length

$\therefore$ ni = current per unit length

In the given problem ni = $\frac{I}{L}$

$\therefore B = \frac{\mu_0 I}{L}$

Flux passing through the circular coil is

$\phi = BS = \left(\frac{\mu_0 I}{L}\right)(\pi r^2)$

Induced emf $e = -\frac{d\phi}{dt} = -\left(\frac{\mu_0 \pi r^2}{L}\right) \cdot \frac{dI}{dt}$

Induced current, $i = \frac{e}{R} = -\left(\frac{\mu_0 \pi r^2}{LR}\right) \cdot \frac{dI}{dt}$

Magnetic moment, M = iA = $i\pi r^2$

or M $= -\left(\frac{\mu_0 \pi^2 r^4}{LR}\right) \cdot \frac{dI}{dt}$ .... (i)

Given, I = I_0cos (300 t)

$\therefore \frac{dI}{dt}$ = - 300I_0 sin (300 t)

Substituting in Eq. (i), We get

$M = \left(\frac{300\pi^2 r^4)}{LR}\right) \mu_0 I_0 \sin(300t)$

$\therefore N = \frac{300\pi^2 r^4}{LR}$

Substituting the value, we get

$N = \frac{300(22/7)^2(0.1)^4}{(10)(0.005)} = 5.926$ or N $\simeq 6$

11. 900

Explanation:

For an adiabatic process,

$PV^\gamma =$ constant or $TV^{\gamma-1} =$ constant

$\therefore \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ or $\frac{T_1}{T_2} = \left(\frac{8V_1}{V_1}\right)^{\gamma-1}$ $(\because V_2 = 8V_1 \text{ (given)})$

$\Rightarrow T_2 = \frac{T_1}{8^{\gamma-1}}$ ...(i)

For a monatomic gas,

$C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R \therefore \gamma = \frac{C_P}{C_V} = \frac{5}{3}$

So, from eqn (i), $T_2 = \frac{T_1}{8^{(5/3-1)}} = \frac{T_1}{4}$

Change in internal energy of the gas,

$\Delta U = nC_V(T_2 - T_1)$

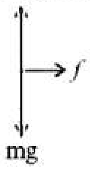
$= 1 \times \frac{3}{2}R \times \left(\frac{T_1}{4} - T_1\right) = \frac{3}{2}R \times \left(\frac{-3T_1}{4}\right)$

$= \frac{3}{2} \times 8 \times \left(\frac{-3}{4}\right) \times 100 = -900\ J$

So, decrease in internal energy of the gas is 900 J.

12. 5

Explanation:



The frictional force is responsible to move the block of mass 1 kg with an acceleration of

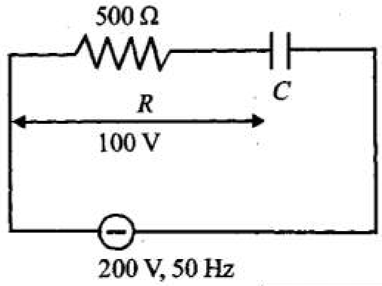
$5\ m/s^2$.

Therefore, frictional force,

$f = m \times a = 1 \times 5 = 5$ N

13. 100.0

Explanation:



From $V_{RMS} = \sqrt{V_C^2 + V_R^2}$

$\Rightarrow V_C^2 + 100^2 = 200^2$

or, $V_C^2 + 10000 = 40000$

$\therefore V_C = 100\sqrt{3}$ V ...(i)

$\tan\phi = \frac{V_C}{V_R} = \frac{100\sqrt{3}}{100}$

$\therefore \phi = 60^0$ ...(ii)

Power consumed, $P = I_{rms} V_{rms} \cos\phi = \frac{1}{2}\frac{V_{rms}^2}{z}$

$\Rightarrow 500 = \frac{200}{z}\frac{1}{2}$

$\Rightarrow z = 40\ \Omega$ ...(iii)

$\cos\phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{40}$

$\therefore R = 20$

And $X_C = \sqrt{z^2 - R^2} = \sqrt{40^2 - 20^2} = 20\sqrt{3}\ \Omega$

$X_C = \frac{1}{C\omega} \Rightarrow 20\sqrt{3} = \frac{1}{C2\pi f}$

$\therefore C = \frac{1}{2\pi f(20\sqrt{3})} = \frac{1}{20\pi\sqrt{3}\times 100}$

$= 10^{-4}$ F $= 100\ \mu$F

14.

(b) P - 3, Q - 1, R - 4, S - 2

Explanation:

If Q_1, Q_2, Q_3 and Q_4 are all positive, then the force will be along + y-direction as components of forces along x-axis cancel out each other.

If Q_1, Q_2 are positive and Q_3, Q_4 are negative the force will act along + x-direction as components of forees along y-axis cancel out each other.

If Q_1, Q_4 are positive and Q_2, Q_3 are negative then attractive force will dominate repulsive force and the force will be along -y direction.

If Q_1, Q_3 positive and Q_2, Q_4 negative components of forces along y-axis cancel out each other. So net force on charge q along x-axis.

15.

(b) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (4), (R) $\rightarrow$ (2), (S) $\rightarrow$ (5)

Explanation:

From force equation, mg - Biℓ = ma

$= \mathrm{mg} - \mathrm{Bi}\ell = \frac{\mathrm{mdv}}{\mathrm{dt}} \Rightarrow \mathrm{mg} - \frac{\mathrm{BBi}\ell}{\mathrm{R}} \times \ell = \frac{\mathrm{mdv}}{\mathrm{dt}}$

$\left[\because \mathrm{i} = \frac{\varepsilon}{\mathrm{R}} = \frac{\mathrm{B}\ell\mathrm{v}}{\mathrm{R}}\right]$

$\Rightarrow \frac{\mathrm{mgR}}{\mathrm{B}^2\ell^2} - \mathrm{v} = \frac{\mathrm{mR}}{\mathrm{B}^2\ell^2}\frac{\mathrm{dv}}{\mathrm{dt}}$

$\frac{\mathrm{B}^2\ell^2}{\mathrm{mR}} \int_{t=0}^{\mathrm{t}} \mathrm{dt} = \int_0^{\mathrm{v}} \frac{\mathrm{dv}}{\frac{\mathrm{mgR}}{\mathrm{B}^2\ell^2} - \mathrm{v}}$

or, $\frac{\mathrm{B}^2\ell^2}{\mathrm{mR}} = \frac{16 \times \frac{1}{16}}{20 \times 10^{-3} \times 10} = \frac{1}{02} = 5$

Now $\frac{\mathrm{mgR}}{\mathrm{B}^2\ell^2} = \frac{20 \times 10^{-3} \times 10 \times 10}{16 \times \frac{1}{16}} = 2$

And $\frac{\mathrm{B}^2\ell^2}{\mathrm{mR}} = \frac{16 \times \frac{1}{16}}{20 \times 10^{-3} \times 10} = \frac{1}{0.2} = 5$

$\therefore 5\mathrm{t} = [-\ell\mathrm{n}(2 - \mathrm{v})]_0^{\mathrm{v}} \Rightarrow -5\mathrm{t} = \ell\mathrm{n}\left[\frac{2-\mathrm{v}}{\mathrm{v}}\right]$

$\therefore$ v = 2 (1 - e^{-5t})

At t = 0.2 sec

v = 2 (1 - $e^{-5 \times 0.2}$)

v = 2 (1 - 0.4)

v = 1.2m /s

At t = 0.2 s

Induced emf $\varepsilon = \mathrm{Bv}\ell$

$\therefore \varepsilon = 4 \times 1.2 \times \frac{1}{4} = 1.2\mathrm{Volt}$

Magnetic force $= \mathrm{BI}\ell \sin\theta = \mathrm{B} \times \frac{\mathrm{B}\ell\mathrm{v}}{\mathrm{R}} \times \ell \times \sin 90^\circ$

$= \frac{4 \times 4 \times \frac{1}{4} \times 1.3 \times \frac{1}{4}}{10} = 0.12\ \mathrm{N}$

Power dissipated as heat $P = i^2R - \frac{v^2}{R}$

$\therefore P = \frac{1.2 \times 1.2}{10} = 0.144\mathrm{watt}$

At terminal velocity, the net force become zero

$\therefore \mathrm{mg} = \mathrm{Bi}\ell \Rightarrow \mathrm{mg} = \mathrm{B} \times \frac{\mathrm{B}\ell\mathrm{v_t}}{\mathrm{R}} \times \ell$

$\therefore \mathrm{v_T} = \frac{\mathrm{mgR}}{\mathrm{B}^2\ell^2} = \frac{20 \times 10^{-3} \times 10 \times 10}{16 \times \frac{1}{16}} = 2\ \mathrm{m/s}$

16.

(c) (A) - II, (B) - I, (C) - IV, (D) - III

Explanation:

$\Delta\mathrm{U} = \mathrm{nC_v}\Delta\mathrm{T}$

For isothermal process T is constant

So, ΔU = 0

A $\longrightarrow$ II

Adiabatic process

$\Delta Q = 0$

$\Delta Q = \Delta U + \Delta W$

$\Delta U = -\Delta W$

Work done by gas is positive

So, ΔU is negative

B $\longrightarrow$ I

For Isochoric process $\Delta W = 0$

C $\longrightarrow$ IV

For Isobaric process

$\Delta W = P\Delta V \neq 0$

$\Delta U = nC_V\Delta T \neq 0$

Heat absorbed goes partly to increase internal energy and partly do work.

Chemistry

17. **(a)** Non-biodegradable pollutant

Explanation:

Non-biodegradable pollutant

18.

(d) 10

Explanation:

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = -54.07 \times 10^3 J - 298 \times 10 J$

$= -57.05 \times 10^3$ J

Also, $\triangle G^O = -2.303\ RT \log K$

$\Rightarrow \log K = \frac{-\Delta G^\circ}{2.303RT}$

$= \frac{57.05 \times 10^3}{5705} = 10$

19.

(d) -3.4 eV

Explanation:

The energy of an electron in a Bohr atom is expressed as

$E_n = -\frac{kZ^2}{n^2}$ where, k = Constant, Z = Atomic number, n = Orbit number

$= -13.6$ eV for H(n = 1)

when n = 2, $E_2 = \frac{-13.6}{2^2}$ eV = -3.40 eV

(n can have only integral value 1, 2, 3,.......∞)

20.

(d) Cathode to anode through internal supply

Explanation:

In electrolytic cell electrolysis occur at the cost of electricity :

At cathode : $M^{n+} + ne \longrightarrow M$ (electron gone in solution)

At anode: $X^{n-} \longrightarrow X + ne^-$ (electron supplied to anode)

Therefore, electron is moving from cathode to anode via internal circuit.

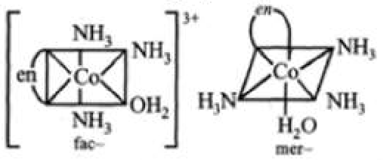
21. **(a)** It will have three geometrical isomers if bidentate 'en' is replaced by two cyanide ligands

(c) It has two geometrical isomers

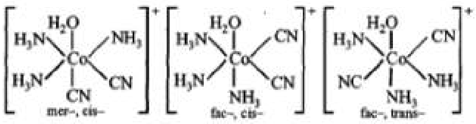
(d) It absorbs light at longer wavelength as compared to $[Co(en)(NH_3)_4]^{3+}$

Explanation:

a. $[Co(en)(NH_3)_3(H_2O)]^{3+}$ has 2 geometrical isomers.



b. Compound $[Co(CN)_2(NH_3)_3(H_2O)]^+$ will have three geometrical isomers.



[Note: fac- and mer- w.r.t. NH_3, cis- and trans- w.r.t. -CN]

c. $[Co(en)(NH_3)_3(H_2O)]^{3+}$ is diamagnetic Due to the presence of strong field ligand 'en' d^6 system (Co^{3+}) forms low spin (l.s) complex in Oh splitting of d-orbitals.

d. $[Co(en)(NH_3)_4]^{3+}$ has larger gap between e_g and t_{2g} than $[Co(en)(NH_3)_3(H_2O)]^{3+}$ So, $[Co(en)(NH_3)_3(H_2O)]^{3+}$ absorbs light at longer wavelength as compared to $[Co(en)(NH_3)_4]^{3+}$.

22. **(a)** the same number of paired electrons

(b) identical arrangement of atoms

(d) nearly the same energy content

Explanation: Resonating structures differ in bonding pattern.

23. **(b)** The ratio of M-X bond length to the cubic unit cell edge length is 0.866.

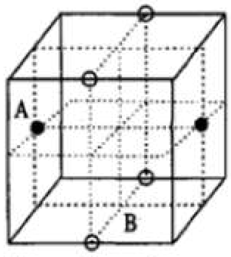
(c) The empirical formula of the compound is MX.

Explanation:

a. Contribution of $M = 2 \times \frac{1}{2} = 1$

Contribution of $X = 4 \times \frac{1}{4} = 1$

∴ Empirical formula is MX



b. Coordination numbers of both M and X is 8.

c. Bond length of M - X bond

$= AB = \sqrt{3} \cdot \frac{a}{2} = 0.866\ a$

d. Assuming anions are in contact, the ratio of ionic radii of cation to anion is 0.732 which is the radius ratio of cubical void.

24. 4.0

Explanation:

PBS, CuS, As_2S_3, CdS are soluble in 50% HNO_3. HgS, Sb_2S_3 are insoluble in 50% HNO_3.

25. 222

Explanation:

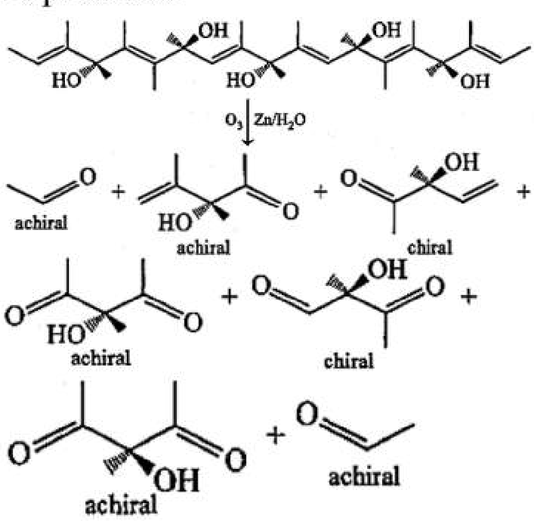
$4(CH_3)_2SiCl_2 + 4H_2O \longrightarrow (CH_3)_8Si_4O_4 + 8HCl$

4 moles of $(CH_3)_2$ $SiCl_2$ produces, 0.75 mol of X.

∴ mass of product 'X' = 0.75 × 296 = 222 g

26. 2.0

Explanation:



27. 9

Explanation:

1 mole solution has 0.1 mole solute and 0.9 mole solvent.

Let M_1 = Molar mass solute

M_2 = Molar mass solvent

Molality, $m = \frac{0.1}{0.9 M_2} \times 1000$ ...(i)

Molarity, M = $\frac{0.1}{0.1M_1+0.9M_2} \times 2 \times 1000$ ...(ii)

$\because$ m = M

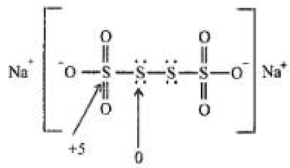
$\Rightarrow \frac{0.1\times1000}{0.9M_2} = \frac{200}{0.1M_1+0.9M_2}$

$\Rightarrow \frac{M_1}{M_2} = 9$

Note: Molarity = $\frac{\text{Moles of solute}}{\text{(Mass of solution/density)}}$

28. 5

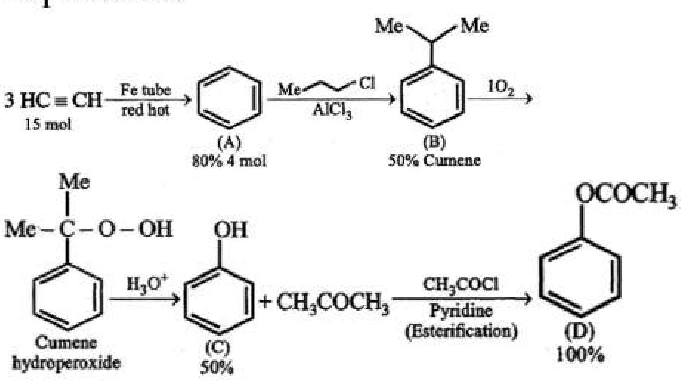
Explanation:



Difference in oxidation number = 5 - 0 = 5

29. 136.0

Explanation:



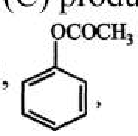
15 mol of C_2H_2 is supposed to form 5 mol of benzene (A) theoretically actual formation of (A) is 80%.

$\therefore$ No. of moles of (A) = $\frac{5\times80}{100}$ = 4 mol

4 moles of (A) produces 50% of (B) = 2 moles of (B).

Further, 2 moles of (B) produces 50% of (C) = 1 mol of (C)

$\therefore$ 1 mole of (C) produces 100% of (D) = 1 mol of (D).

M.W. of (D), , acetophenone = 136

$\therefore$ The amount of the product D formed is 136 g.

30. **(a)** A - III, B - II, C - IV, D - I

Explanation:

$B_2H_6 \Rightarrow$ electron deficient hydride

HF $\Rightarrow$ electron rich hydride

$CH_4 \Rightarrow$ electron rich hydride

$MgH_2 \Rightarrow$ Saline hydride

31.

(d) (A) - (V), (B) - (I), (C) - (II), (D) - (III)

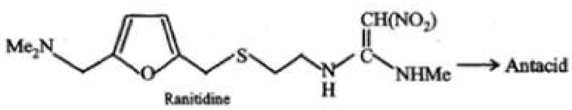
Explanation:

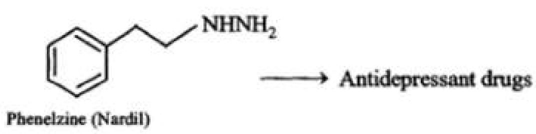
Name of oxo acids	Oxidation state
Hypophosphoroiis acid (H_3PO_2)	+1
Orthophosphoroiis acid (H_3PO_3)	+3
Hypophosphorit: acid ($H_4P_2O_6$)	+4
Orthophosphoric acid (H_3PO_4)	+5

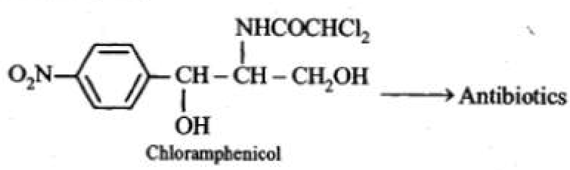
32.

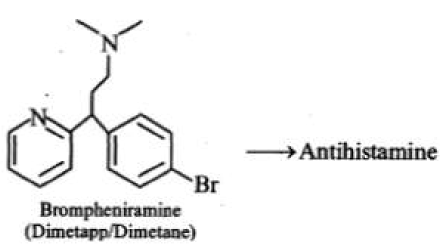
(c) (i) - (D); (ii) - (A); (iii) - (B); (iv) - (C)

Explanation:









Maths

33.

(c) $\frac{-7}{24}$

Explanation:

$\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$

$2\cos^{-1}\frac{2}{\sqrt{5}} = 2\tan^{-1}\frac{1}{2} = \tan^{-1}\frac{2\cdot\frac{1}{2}}{1-\frac{1}{4}} = \tan^{-1}\frac{4}{3}$

Now, $\tan\left(\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{4}{3}\right) = \tan\left(\tan^{-1}\frac{\frac{3}{4}-\frac{4}{3}}{1+\frac{3}{4}\cdot\frac{4}{3}}\right)$

$= \tan\left(\tan^{-1}\left(\frac{-7}{24}\right)\right) = \frac{-7}{24}$

34.

(c) a = 2, b = 3

Explanation:

PQRS is a parallelogram if and only if the mid point of the diagonals PR is same as that of the mid-point of QS. That is, if and only if $\frac{1+5}{2} = \frac{4+a}{2}$ and $\frac{2+7}{2} = \frac{6+b}{2}$

$\Rightarrow$ a = 2 and b = 3

35.

(d) $\frac{6}{11}$

Explanation:

Sample space $\rightarrow$ $^{12}C_2$

Number of possibilities for z is even.

$z = 0 \Rightarrow^{11} C_1$

$z = 2 \Rightarrow^{9} C_1$

$z = 4 \Rightarrow^{7} C_1$

$z = 6 \Rightarrow^{5} C_1$

$z = 8 \Rightarrow^{3} C_1$

$z = 10 \Rightarrow^{1} C_1$

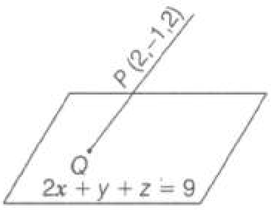
Total = 36

$\therefore$ Probability $= \frac{36}{66} = \frac{6}{11}$

36. **(a)** $\sqrt{3}$

Explanation:

Since, l = m = n = $\frac{1}{\sqrt{3}}$



$\therefore$ Equations of line are $\frac{x-2}{1/\sqrt{3}} = \frac{y+1}{1/\sqrt{3}} = \frac{z-2}{1/\sqrt{3}}$

$\Rightarrow$ x - 2 = y + 1 = z - 2 = r [say]

$\therefore$ Any point on the line is

Q $\equiv$ (r + 2, r - 1, r + 2)

$\because$ Q lies on the plane 2x + y + z = 9

$\therefore$ 2(r + 2) + (r - 1) + (r + 2)

$\Rightarrow$ 4r + 5 = 9 $\Rightarrow$ r = 1

$\Rightarrow$ Q(3, 0, 3)

$\therefore$ PQ = $\sqrt{(3-2)^2 + (0+1)^2 + (3-2)^2} = \sqrt{3}$

37. **(c)** f(x) has three real roots if -4 < a < 4

(d) f(x) has only real root if a > 4

Explanation: f(x) = x^5 - 5x + a

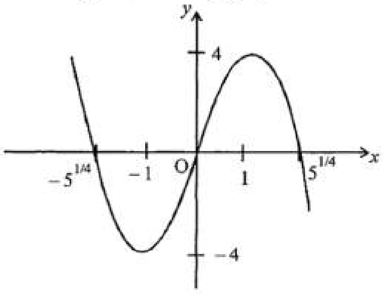
$f(x) = 0 \Rightarrow x^5 - 5x + a = 0 \Rightarrow a = 5x - x^5 = g(x)$

$\Rightarrow g(x) = 0$ when $x = 0, 5^{\frac{1}{4}}, -5^{\frac{1}{4}}$

and $g'(x) = 0 \Rightarrow x = 1, -1$

Also $g(-1) = -4$ and $g(1) = 4$

Thus graph of g(x) will be as shown below.



From graph, it is clear that if $a \in (-4, 4)$

then $g(x) = a$ or $f(x) = 0$ has 3 real roots

If $a > 4$ or $a < -4$

then $f(x) = 0$ has only one real root.

$\therefore$ option f(x) has only real root if $a > 4$ and f(x) has three real roots if $-4 < a < 4$ are the correct options.

38. **(b)** h is increasing whenever f s increasing

(c) h is decreasing whenever f is decreasing

Explanation: $h(x) = f(x) - (f(x))^2 + (f(x))^3 \quad \forall x \in R$

$h'(x) = f'(x)[1 - 2f(x) + 3(f(x))^2]$

$= 3f'(x)\left[(f(x))^2 - \frac{2}{3}f(x) + \frac{1}{3}\right]$

$= 3f'(x)\left[\{f(x) - \frac{1}{3}\}^2 + \frac{2}{9}\right]$

Here $h'(x) < 0$ whenever $f'(x) < 0$ and $h'(x) > 0$ whenever $f'(x) > 0$.

Hence h(x) increases (decreases) whenever f(x) increases (decreases).

39. **(a)** $b_0 = 0, b_1 = n$

Explanation: Putting $\theta = 0$, we get $b_0 = 0$

$\therefore \sin n\theta = \sum_{r=1}^{n} b_r \sin^r\theta \Rightarrow \frac{\sin n\theta}{\sin\theta} = \sum_{r=1}^{n} b_r(\sin\theta)^{r-1}$

$= b_1 + b_2 \sin\theta + b_3 \sin^2\theta + \ldots + b_n \sin^{n-1}\theta$

$\therefore \lim_{\theta\to 0} \frac{\sin n\theta}{\sin\theta} = b_1 \Rightarrow b_1 = n$

40. 0.5

Explanation:

Let $X = \frac{4z^2+3z+2}{4z^2-3z+2}$

It can be written as

$= 1 + \frac{6z}{4z^2-3z+2}$

Now $X = 1 + \frac{6}{2\left(2z+\frac{1}{z}\right)-3}$

$\because X \in R$, then $2z + \frac{1}{z} \in R$

$\Rightarrow 2z + \frac{1}{z} = 2\bar{z} + \frac{1}{\bar{z}} \Rightarrow 2(z - \bar{z}) - \frac{z-\bar{z}}{|z|^2} = 0$

$\because (z - \bar{z})\left(2 - \frac{1}{|z|^2}\right) = 0$

$\because z \neq \bar{z}$ (given). So, $|z|^2 = \frac{1}{2} = 0.5$

41. 2.0

Explanation:

Given that

$c = \sqrt{23}$; a = 3; b = 4

We have cot A = $\frac{\cos A}{\sin A} = \frac{b^2+c^2-a^2}{2bc \sin A}$

$= \frac{b^2+c^2-a^2}{2.2\triangle} \left\{\triangle = \frac{1}{2} bc \sin A\right\}$

$\therefore$ Cot A $= \frac{b^2+c^2-a^2}{4\triangle}$

Similarly, cot B $= \frac{a^2+c^2-b^2}{4\triangle}$ & cot C $= \frac{a^2+b^2-c^2}{4\triangle}$

Now $\frac{\cot A+\cot C}{\cot B} = \frac{b^2+c^2-a^2+a^2+b^2-c^2}{a^2+c^2-b^2}$

$= \frac{2b^2}{a^2+c^2-b^2} = \frac{2(16)}{9+23-16} = \frac{32}{16} = 2$

42. 569.0

Explanation:

Counting integers starting from 2

Case I: At unit's place we can fill 2/3/4/6/7

i.e., 2 0 **2** [5] $\rightarrow$ 5 ways

At unit's place and ten's place we can fill digits as 3/4/6/7 and 0/2/3/4/6/7

or 2 0 [4] [6] $\rightarrow$ 24 ways

(Numbers except 0 or 2 in 3rd place)

Case II: If non-zero number on 2nd place

i.e., 2 [5] [6] [6] = 180 ways

Counting integers starting from 3

3 [6] [6] [6] = 216 ways

Counting integer starting from 4

Case I: If 0, 2 or 3 on 2nd place

i.e., 4 [3] [6] [6]

= 108 ways

Case II: If 4 on 2nd place

i.e., 4 4 [6] [6] = 36 ways

$\therefore$ Total 5 + 24 + 180 + 216 + 108 + 36 = 569 numbers

43. 7

Explanation:

Given, n > 3 ∈ Integer

and $\frac{1}{\sin\left(\frac{\pi}{n}\right)}=\frac{1}{\sin\left(\frac{2\pi}{n}\right)}+\frac{1}{\sin\left(\frac{3\pi}{n}\right)}$

$\Rightarrow \frac{1}{\sin\frac{\pi}{n}}-\frac{1}{\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$

$\Rightarrow \frac{\sin\frac{3\pi}{n}-\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\cdot\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$

$\Rightarrow 2\cos\left(\frac{2\pi}{n}\right)\cdot\sin\frac{\pi}{n}=\frac{\sin\frac{\pi}{n}\cdot\sin\frac{3\pi}{n}}{\sin\frac{2\pi}{n}}$

$\Rightarrow 2\sin\frac{2\pi}{n}\cdot\cos\frac{2\pi}{n}=\sin\frac{3\pi}{n}$

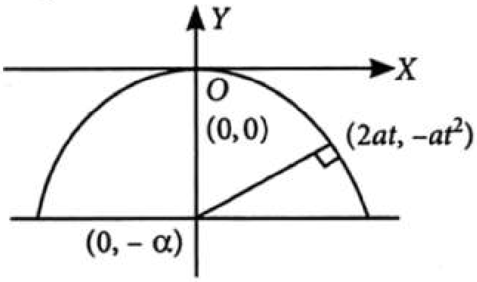
$\Rightarrow \sin\frac{4\pi}{n}=\sin\frac{3\pi}{n}$

$\Rightarrow \frac{4\pi}{n}=\pi-\frac{3\pi}{n}$

$\Rightarrow \frac{7\pi}{n}=\pi\Rightarrow$ n = 7

44. 12

Explanation:



Slope of normal $=\frac{1}{t}=\frac{1}{\sqrt{6}}\Rightarrow t=\sqrt{6}$

Now, $\frac{-at^2+\alpha}{2at}=\frac{1}{t}$ (Given)

$-at^2+\alpha=2a\Rightarrow -6a+\alpha=2a$

$\therefore \alpha=8a$

To determine points A and B, we have

$x^2=-4a\cdot(-8a)\Rightarrow x^2=32a^2$

$\therefore x=4\sqrt{2}a,-4\sqrt{2}a$

Thus, $A(-4\sqrt{2}a,-8a)$ and $B(4\sqrt{2}a,-8a)$

$AB^2=(8\sqrt{2}a)^2=128a^2=s$

Length of latus rectum, $r=4a$

$\frac{r}{s}=\frac{4a}{128a^2}=\frac{1}{16}$ (Given)

$\therefore a=\frac{1}{2}$, and then $24a=12$

45. 4

Explanation:

$$|A| = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 2\sqrt{k} & 1+2k & -2k \\ -2\sqrt{k} & 1+2k & -1 \end{vmatrix} [C_2 \to C_2 - C_3]$$

$$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 4\sqrt{k} & 0 & 1-2k \\ -2\sqrt{k} & 1+2k & -1 \end{vmatrix} [R_2 \to R_2 - R_3]$$

$= (1 + 2k)(8k - 4k + 4k^2 + 1) = (2k + 1)^3$

Since B is skew symmetric of odd order,

$\therefore |B| = 0$

Hence, $|\text{Adj } A| + |\text{Adj } B| = |A|^2 + |B|^2 = 10^6$

$\Rightarrow (2k + 1)^6 = 10^6 \Rightarrow k = 4.5, \therefore [k] = 4$

46.

(b) $(P) \to (2), (Q) \to (4), (R) \to (3), (S) \to (5)$

Explanation:

Given α and β are roots of $x^2 + x - 1 = 0$

$\Rightarrow \alpha + \beta = -1, \alpha\beta = -1$

P. $M = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$

Row 1 can be arranged in 3! ways and correspondings the other two rows can be arranged in 2 ways.

$\therefore$ Total number of ways $= 3! \times 2 = 12$

Q. Let $M = \begin{bmatrix} p & m & n \\ m & q & t \\ n & t & r \end{bmatrix}$

Let $m = \alpha, n = \beta, t = 1$

One such arrangement

So m, n, r can be arranged in 3! ways and the remaining in 1 way.

$\therefore$ Required total number of ways $= 3! \times 1 = 6$

R. $\begin{bmatrix} 0 & m & n \\ -m & 0 & t \\ -n & -t & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} m \\ 0 \\ -t \end{bmatrix}$

We have $D = D_x = D_y = D_z = 0$

Infinite many solutions

S. We have $M=\begin{bmatrix}1 & \alpha & \beta \\ \beta & \alpha & 1 \\ \alpha & 1 & \beta\end{bmatrix}$

$|M|=\alpha\beta+\alpha^2+\beta^2-1-\alpha\beta^2-\alpha^2\beta$

$=\alpha\beta+(\alpha+\beta)^2-2\alpha\beta-1-\alpha\beta(\alpha+\beta)$

$=(\alpha+\beta)^2-\alpha\beta-1-\alpha\beta(\alpha+\beta)$

$=1+1-1-(-1)(-1)=1+1-1-1=0$

47.

(c) P → 3, Q → 4, R → 1, S → 2

Explanation:

P. Given that $[\vec{a}\ \ \vec{b}\ \ \vec{c}]=2$

$\therefore [2(\vec{a}\times\vec{b})3(\vec{b}\times\vec{c})\vec{c}\times\vec{a}]$

$=6[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$

$=6[\vec{a}\ \ \vec{b}\ \ \vec{c}]^2=6\times4=24$

$\therefore$ (P) → (3)

Q. Given that $[\vec{a}\vec{b}\vec{c}]=5$

$\therefore [3(\vec{a}+\vec{b})\vec{b}+\vec{c}\ \ 2(\vec{c}+\vec{a})]$

$=6[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]$

$=6\times2[\vec{a}\ \ \vec{b}\ \ \vec{c}]=6\times2\times5=60$

$\therefore$ (Q) → (4)

R. Given that $\frac{1}{2}|\vec{a}\times\vec{b}|=20\Rightarrow|\vec{a}\times\vec{b}|=40$

$\therefore \frac{1}{2}|(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})|=\frac{1}{2}|-2\vec{a}\times\vec{b}+3\vec{b}\times\vec{a}|$

$=\frac{1}{2}\times5|\vec{a}\times\vec{b}|=\frac{5}{2}\times40=100$

$\therefore$ (R) → (1)

S. Given that $|\vec{a}\times\vec{b}|=30$

$\therefore |(\vec{a}+\vec{b})\times\vec{a}|=|(\vec{b}\times\vec{a})|=30$

$\therefore$ (S) → (2)

48.

(d) (P) - (2), (Q) - (3), (R) - (1), (S) - (4)

Explanation:

P(2) Let $f(x)=ax^2+bx+c$

where a, b, c ≥ 0 and a, b, c are integers.

$\because f(0)=0\Rightarrow c=0$

$\therefore f(x)=ax^2+bx$

Also $\int_0^1 f(x)dx=1$

$\Rightarrow\left[\frac{ax^3}{3}+\frac{bx^2}{2}\right]_0^1=1\Rightarrow\frac{a}{3}+\frac{b}{2}=1\Rightarrow 2a+3b=6$

$\because$ a and b are integers

$\because$ a = 0 and b = 2

or a = 3 and b = 0

$\therefore$ There are only 2 solutions.

$Q(3) f(x)=\sin x^2+\cos x^2$

f(x) is max. $\sqrt{2} a t x^2=\frac{\pi}{4} or \frac{9 \pi}{4}$

$\Rightarrow x=\pm \frac{\sqrt{\pi}}{2}$ or $\pm \frac{3 \sqrt{\pi}}{2} \in[-\sqrt{13}, \sqrt{13}]$

$\therefore$ There are four points.

$R(1) I=\int_{-2}^2 \frac{3 x^2}{1+e^x} d x=\int_{-2}^2 \frac{3 x^2}{1+e^{-x}} d x\left[\text{Using} \int_a^b f(x) d x=\int_a^b f(a+b-x) d x\right]$

$=\int_{-2}^2 \frac{3 x^2 e^x}{1+e^x} d x$

$2 \mathrm{I}=\int_{-2}^2 \frac{3 x^2\left(1+e^x\right)}{1+e^x} d x=\int_{-2}^2 3 x^2 d x$

$2 I=\left(x^3\right)_{-2}^2=8-(-8)=16 \Rightarrow I=8$

$S(4) \frac{\int_{-1/2}^{1/2} \cos 2 x \log \left(\frac{1+x}{1-x}\right) d x}{\int_0^{1/2} \cos 2 x \log \left(\frac{1+x}{1-x}\right) d x}=0$

$\because$ Numerator =0, function being odd.

Hence option ((P) - (2), (Q) - (3), (R) - (1), (S) - (4)) is correct sequence.